Corrected Status of the Robin–MVDC Programme
Superseding the Previous Robin–Mertens Proof Branch and Consolidated Manuscript

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June 1, 2026

Abstract

This manuscript is a corrected status version of the Robin–MVDC programme. It supersedes the previous Robin–Mertens compensation branch and the previous consolidated manuscript as proof texts. It is not presented as a completed proof of Robin's inequality or of the Riemann Hypothesis.

The logarithmic bridge, namely the step giving $P(N) < \log N$ for a least hypothetical Robin counterexample N, where $P(N)$ denotes the largest prime factor of N, is kept as an independent auxiliary result. The withdrawn part is the former Mertens compensation argument based on an abstract J_1-tail in a lowered $B(n)$-envelope. That argument did not certify that the lowered product remained an upper envelope for $\sigma(n)/n$.

The corrected state is reduced to two active analytic routes. The first is a signed Chebyshev/MVDC-centre route, which preserves the cancellation in the beta error but requires a one-sided theorem for the signed weighted average of $t - \theta(t)$. The second is a first-moment reciprocal-prime route, which avoids the Chebyshev oscillation but requires a sharper local or cumulative estimate for a block sum $\sum_{Y<p\leq x} 1/(p-1)$ than the presently available endpoint estimates provide.

1 Status of This Version

This version has a deliberately narrow purpose. It records what remains after removing the invalid proof step from the earlier Robin–Mertens route. It should be read as a corrected research programme, not as a final proof.

The following status distinctions are important.

- The logarithmic bridge remains a separate auxiliary result. Its role is to justify the use of $P(N) < \log N$ for a least hypothetical Robin counterexample $N > 5040$.
- The former Mertens compensation branch is withdrawn as a proof of the final Robin step.
- The former consolidated manuscript is superseded as a proof text, because it inherited the same $B(n)$-envelope gap.
- The present manuscript keeps only the corrected ledger identities and states the remaining analytic targets explicitly.

For orientation, the corrected material is organised below in a self-contained form. First we record the exact Robin ledger and isolate why the previous B_J-envelope subtraction was not certified. Then Route I introduces the endpoint-normalised W-error and the signed Chebyshev/MVDC centre. Route II avoids that oscillating centre and rewrites the beta block as a first-moment reciprocal-prime certificate. The final section states the remaining analytic targets explicitly.

2 Exact Robin Ledger

Let

$$\beta(x) = \prod_{p \le x} \frac{p}{p-1}, \qquad E(x) = \log \beta(x) - \gamma - \log\log x.$$

For the extremal candidate profiles used below, the support is full up to $x = P(n)$:

$$n = \prod_{p \le x} p^{a_p}, \qquad a_p \ge 1.$$

For a completely arbitrary integer one may instead put $a_p = 0$ for missing primes and keep the same product over all $p \le x$. In the full-support case,

$$\frac{\sigma(n)}{n} = \beta(x) \prod_{p \le x} \left(1 - p^{-(a_p+1)}\right).$$

Define the actual multiplicative deficit

$$A(n) := -\sum_{p \le x} \log \left(1 - p^{-(a_p+1)}\right),$$

and the logarithmic reserve

$$B_{\log}(n) := \log \frac{\log\log n}{\log x}.$$

Then

$$\log \frac{\sigma(n)}{n} = \gamma + \log\log x + E(x) - A(n).$$

Therefore Robin's inequality for n follows from the exact gate

$$E(x) \le A(n) + B_{\log}(n). \tag{1}$$

Derivation of the gate

For a prime power,

$$\frac{\sigma(p^a)}{p^a} = 1 + \frac{1}{p} + \cdots + \frac{1}{p^a} = \frac{1 - p^{-(a+1)}}{1 - p^{-1}} = \frac{p}{p-1} \left(1 - p^{-(a+1)}\right).$$

Multiplying over all primes in the full support gives the product above. By definition,

$$\log \beta(x) = \gamma + \log\log x + E(x), \qquad \sum_{p \le x} \log \left(1 - p^{-(a_p+1)}\right) = -A(n).$$

Hence

$$\log \frac{\sigma(n)}{n} = \gamma + \log\log x + E(x) - A(n).$$

On the other hand,

$$\log(e^\gamma \log\log n) = \gamma + \log\log x + \log \frac{\log\log n}{\log x} = \gamma + \log\log x + B_{\log}(n).$$

Therefore

$$\log \frac{\sigma(n)}{n} \le \log(e^\gamma \log\log n)$$

is implied exactly by (1).

Candidate and reserve convention

Below, a CA profile means the standard extremal candidate profile retained after the Robin/Alaoglu–Erdos reduction to record-setting divisor-sum profiles. The manuscript does not reprove that reduction; it uses it to name the candidate profile on which the remaining ledger is tested. For such a profile with largest prime x, write the corresponding integer as

$$n_{\mathrm{CA}}(x) = \prod_{p \le x} p^{a_p(x)}, \qquad a_p(x) \ge 1.$$

The CA deficit and the raw logarithmic reserve are

$$A_{\mathrm{CA}}(x) := A(n_{\mathrm{CA}}(x)), \qquad B_{\log,\mathrm{CA}}(x) := \log \frac{\log\log n_{\mathrm{CA}}(x)}{\log x}.$$

Thus the raw CA reserve against the beta error $E(x)$ is

$$R^E_{\mathrm{CA}}(x) := A_{\mathrm{CA}}(x) + B_{\log,\mathrm{CA}}(x).$$

The phrase CA support block means an interval $(x_{i-1}, x_i]$ between two successive sampled CA support endpoints. In a proof ledger one may replace $R^E_{\mathrm{CA}}(x_i)$ by any certified lower bound for it.

3 Why the Previous $B(n)$-Envelope Step Was Invalid

The withdrawn branch introduced a lowered model envelope of the form

$$B_J(n) = \beta(x) \prod_{p \in J_1(n)} \left(1 - \frac{1}{p^2}\right)$$

and tried to use $\beta(x) - B_J(n)$ to compensate the positive surplus in an explicit Mertens-type upper bound for $\beta(x)$. Here $J_1(n)$ denotes the chosen unit-exponent prime tail in that withdrawn construction.

The algebraic subtraction is harmless only if $B_J(n)$ is still a certified upper envelope:

$$\prod_{p \le x} \left(1 - p^{-(a_p+1)}\right) \le \prod_{p \in J_1(n)} \left(1 - \frac{1}{p^2}\right). \tag{2}$$

Without (2), the chosen abstract tail may make $B_J(n)$ smaller than the true value of $\sigma(n)/n$. In that case the quantity $\beta(x) - B_J(n)$ is not an available proof reserve.

Thus the corrected route must use either the actual CA-profile deficit, or a separate certified envelope which is proved to dominate $\sigma(n)/n$.

4 Route I: Signed Chebyshev/MVDC Centre

The first active route keeps the cancellation in the beta error instead of separating it into coarse positive endpoint bounds. Let

$$\theta(x) = \sum_{p \le x} \log p, \qquad r(x) = \theta(x) - x.$$

Define the endpoint-normalised beta error

$$W(x) := E(x) - \frac{r(x)}{x \log x}.$$

For the same CA profile, write

$$H_{\mathrm{CA}}(x) := \log n_{\mathrm{CA}}(x) - \theta(x).$$

Also put

$$\mathcal{B}_2(x) := B_{\log,\mathrm{CA}}(x) - \frac{r(x) + H_{\mathrm{CA}}(x)}{x\log x}.$$

Then the exact CA ledger gap

$$G_{\mathrm{CA}}(x) := A_{\mathrm{CA}}(x) + B_{\log,\mathrm{CA}}(x) - E(x)$$

can be rewritten as

$$G_{\mathrm{CA}}(x) = A_{\mathrm{CA}}(x) + \frac{H_{\mathrm{CA}}(x)}{x\log x} + \mathcal{B}_2(x) - W(x). \tag{3}$$

Indeed,

$$B_{\log,\mathrm{CA}}(x) = \frac{r(x) + H_{\mathrm{CA}}(x)}{x\log x} + \mathcal{B}_2(x), \qquad E(x) = \frac{r(x)}{x\log x} + W(x),$$

so the endpoint term $r(x)/(x\log x)$ cancels exactly. This is why the corrected beta side propagates $W(x)$, not the raw $E(x)$. The endpoint-corrected CA reserve compared with $W(x)$ is therefore

$$R^W_{\mathrm{CA}}(x) := A_{\mathrm{CA}}(x) + \frac{H_{\mathrm{CA}}(x)}{x\log x} + \mathcal{B}_2(x).$$

The condition $W(x) \le R^W_{\mathrm{CA}}(x)$ is exactly the CA version of the gate (1), written after endpoint cancellation.

For $2 < Y < x$, define the renormalised beta block

$$Q(Y,x) = \sum_{Y<p\le x} -\log(1-1/p) - \log\frac{\log x}{\log Y}.$$

Then

$$E(x) = E(Y) + Q(Y,x).$$

Writing

$$\rho(x) = \frac{\theta(x) - x}{x\log x}, \qquad W(x) = E(x) - \rho(x),$$

one obtains the propagated W-block

$$W(x) = W(Y) + \widetilde{Q}(Y,x), \qquad \widetilde{Q}(Y,x) := Q(Y,x) - (\rho(x) - \rho(Y)).$$

On a block $2 < Y < x$, introduce

$$\omega(t) = \frac{\log t + 1}{t^2(\log t)^2}.$$

The corrected block identity is obtained by splitting $-\log(1-1/p) = 1/p + \sum_{m\ge2} 1/(mp^m)$ and applying partial summation to the first-order part; it has the signed form

$$\widetilde{Q}(Y,x) = \int_Y^x (\theta(t) - t)\omega(t)\,dt + \mathcal{T}_2(Y,x), \tag{4}$$

where

$$\mathcal{T}_2(Y,x) := \sum_{Y<p\le x}\sum_{m\ge2}\frac{1}{mp^m}$$

is the explicitly bounded second-order product tail. For completeness, the first-order calculation is as follows. Since

$$\frac{d}{dt}\left(\frac{1}{t\log t}\right) = -\omega(t),$$

partial summation gives

$$\sum_{Y<p\le x}\frac{1}{p}=\frac{\theta(x)}{x\log x}-\frac{\theta(Y)}{Y\log Y}+\int_Y^x\theta(t)\omega(t)\,dt.$$

Subtracting $\log(\log x/\log Y)$ and the endpoint correction $\rho(x)-\rho(Y)$ leaves exactly

$$\int_Y^x(\theta(t)-t)\omega(t)\,dt,$$

because

$$\int_Y^x t\omega(t)\,dt=\log\frac{\log x}{\log Y}-\frac{1}{\log x}+\frac{1}{\log Y}.$$

The natural MVDC centre of this signed block is

$$K(Y,x)=\frac{-\int_Y^x(\theta(t)-t)\omega(t)\,dt}{\int_Y^x\sqrt{t}\,\omega(t)\,dt}. \tag{5}$$

Indeed, if

$$W_0(Y,x):=\int_Y^x\sqrt{t}\,\omega(t)\,dt,$$

then (5) is exactly the normalisation

$$\int_Y^x(\theta(t)-t)\omega(t)\,dt=-K(Y,x)W_0(Y,x),$$

so that

$$\widetilde{Q}(Y,x)=-K(Y,x)W_0(Y,x)+\mathcal{T}_2(Y,x).$$

Thus the proof obligation becomes a one-sided lower bound for the signed centre $K(Y,x)$, not a coarse absolute bound for $|\theta(t)-t|$. For a CA support block $x_{i-1}<x_i$, let U_{i-1}^W be a certified upper bound for $W(x_{i-1})$, and let R_i^W be a certified lower bound for $R_{\mathrm{CA}}^W(x_i)$. Inserting the block identity gives

$$W(x_i)\le U_{i-1}^W-K(x_{i-1},x_i)W_0(x_{i-1},x_i)+\mathcal{T}_2(x_{i-1},x_i).$$

Hence $W(x_i)\le R_i^W$ follows if

$$K(x_{i-1},x_i)\ge K_i^{\mathrm{req}}:=\frac{U_{i-1}^W+\mathcal{T}_2(x_{i-1},x_i)-R_i^W}{W_0(x_{i-1},x_i)}.$$

The missing theorem in this route is:

Analytic Target 1 (Signed Chebyshev/MVDC centre)**.** *Prove, for all sufficiently large CA support blocks,*

$$K(x_{i-1},x_i)\ge K_i^{\mathrm{req}}.$$

The finite range below the first asymptotic block is to be certified computationally.

This route is numerically strong because it preserves the sign and the cancellation of the beta error. Its obstruction is also clear: the proof requires a one-sided theorem for the signed weighted average of $t-\theta(t)$ (equivalently, for the negative weighted average of $\theta(t)-t$). Standard absolute estimates for $|\theta(t)-t|$ do not close this route, because they destroy the sign.

5 Route II: First-Moment Reciprocal-Prime Certificate

The second active route avoids the Chebyshev oscillation and works directly with a first MVDC moment for the beta block. For $2 < Y < x$, define

$$Q(Y,x) = \sum_{Y<p\le x} -\log(1-1/p) - \log\frac{\log x}{\log Y}.$$

Put

$$\nu = \pi(x) - \pi(Y), \qquad H = \log\frac{\log x}{\log Y}, \qquad \mu = \frac{H}{\nu},$$

and

$$u_p = e^{-\mu}\frac{p}{p-1} - 1.$$

This choice removes the block main term evenly, because $H = \mu\nu$. For each prime in the block,

$$\log(1+u_p) = \log\left(e^{-\mu}\frac{p}{p-1}\right) = -\mu - \log(1-1/p).$$

Summing over the ν primes gives

$$\sum_{Y<p\le x} \log(1+u_p) = \sum_{Y<p\le x} -\log(1-1/p) - \mu\nu = Q(Y,x).$$

Since $1+u_p > 0$ and $\log(1+u_p) \le u_p$, we get

$$Q(Y,x) = \sum_{Y<p\le x} \log(1+u_p) \le M_1(Y,x) := \sum_{Y<p\le x} u_p.$$

Writing

$$S_{-1}(Y,x) := \sum_{Y<p\le x} \frac{1}{p-1},$$

the first moment has the exact form

$$M_1(Y,x) = e^{-\mu}\big(\nu + S_{-1}(Y,x)\big) - \nu. \tag{6}$$

This follows from

$$u_p = (e^{-\mu}-1) + \frac{e^{-\mu}}{p-1}.$$

Let U_i be the cumulative upper ledger for $E(x_i)$, and let R_i be a certified lower bound for the raw CA reserve $R^E_{\mathrm{CA}}(x_i)$. The route closes if

$$U_i \le R_i.$$

Equivalently, on the block $(x_{i-1}, x_i]$, put

$$\nu_i = \pi(x_i) - \pi(x_{i-1}), \qquad H_i = \log\frac{\log x_i}{\log x_{i-1}}, \qquad \mu_i = \frac{H_i}{\nu_i}.$$

It is enough to prove

Analytic Target 2 (First-moment S_{-1} certificate)**.**

$$S_{-1}(x_{i-1}, x_i) \le \nu_i(e^{\mu_i} - 1) + e^{\mu_i}(R_i - U_{i-1}).$$

The equivalence is only algebra. Since

$$E(x_i) = E(x_{i-1}) + Q(x_{i-1}, x_i) \le U_{i-1} + M_1(x_{i-1}, x_i),$$

it is enough to prove

$$M_1(x_{i-1}, x_i) \le R_i - U_{i-1}.$$

Substituting

$$M_1 = e^{-\mu_i}(\nu_i + S_{-1}) - \nu_i$$

and solving for S_{-1} gives exactly the displayed target inequality.

This route is algebraically cleaner than Route I because no Chebyshev oscillation appears. Its obstruction is that known global endpoint estimates for reciprocal-prime sums are too coarse when inserted independently at Y and x. The missing input must be a sharper local or cumulative estimate for the block sum $S_{-1}(Y, x)$.

6 What MVDC Currently Provides

The standalone MVDC papers [6, 7] show that centre decomposition is very effective for smooth products and for numerical Euler-product corrections. In the prime Euler-product setting, however, the MVDC centre and the higher moments still involve sums over primes. Therefore MVDC does not yet provide a closed analytic proof by itself.

In the present Robin programme, MVDC should be understood as the organising mechanism which identifies the correct centred objects:

$$K(Y, x) \quad \text{in Route I}, \qquad M_1(Y, x) \quad \text{in Route II}.$$

The remaining mathematical work is to prove one of the corresponding one-sided analytic targets.

7 Numerical Evidence and Its Role

The numerical audits in the repository check the algebraic bookkeeping and the scale of the two targets. They do not replace the missing analytic theorem. The auxiliary scripts and generated audit tables are maintained in the public support repository [5].

block	K_i	K_i^{req}	margin
3 329 267 → 6 382 007	1.089125	0.143480	+0.945645
6 382 007 → 12 253 883	1.082018	0.135441	+0.946577
12 253 883 → 29 093 377	1.065904	0.392437	+0.673467
29 093 377 → 56 048 351	1.040017	0.108140	+0.931877

block	$M_1(Y, x)$	$U(x) - E(x)$	$R(x) - U(x)$
3 329 267 → 6 382 007	$-2.899 \cdot 10^{-5}$	$1.70 \cdot 10^{-10}$	$2.001 \cdot 10^{-5}$
6 382 007 → 12 253 883	$-2.389 \cdot 10^{-6}$	$2.67 \cdot 10^{-10}$	$1.387 \cdot 10^{-5}$
12 253 883 → 29 093 377	$-8.065 \cdot 10^{-6}$	$2.88 \cdot 10^{-10}$	$8.561 \cdot 10^{-6}$
29 093 377 → 56 048 351	$-3.795 \cdot 10^{-6}$	$1.84 \cdot 10^{-10}$	$5.837 \cdot 10^{-6}$
56 048 351 → 99 999 989	$-1.186 \cdot 10^{-6}$	$4.53 \cdot 10^{-10}$	$4.317 \cdot 10^{-6}$

The first table supports Route I. The second supports Route II. In both cases the missing step is not numerical but analytic.

8 Conclusion

The previous final proof claim is withdrawn. The corrected status is:

the logarithmic bridge remains auxiliary,
the former compensation branch and consolidated proof are superseded.

The Robin–MVDC programme now has two explicit analytic targets:

$$K_i \geq K_i^{\mathrm{req}} \qquad \text{or} \qquad S_{-1}(x_{i-1}, x_i) \leq \nu_i(e^{\mu_i} - 1) + e^{\mu_i}(R_i - U_{i-1}).$$

Proving either target in the required infinite CA range would reopen the Robin route. Until such a theorem is proved, the programme remains an open research direction rather than a completed proof of Robin's inequality or of the Riemann Hypothesis.

References

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